

Dynamics of M-theory vacua

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February 1, 2008

Abstract

At very early times, the universe was not in a vacuum state. Under the assumption that the deviation from equilibrium was large, in particular that it is higher than the scale of inflation, we analyse the conditions for local transitions between states that are related to different vacua. All pathways lead to an attractor solution of a description of the universe by eternal inflation with domains that have different low energy parameters. The generic case favors transitions between states that have significantly different parameters rather than jumps between nearby states in parameter space. I argue that the strong CP problem presents a potential difficulty for this picture, more difficult than the hierarchy problem or the cosmological constant problem. Finally, I describe how the spectrum of quark masses may be a probe of the early dynamics of vacuum states. As an example, by specializing to the case of intersecting braneworld models, I show that the observed mass spectrum, which is approximately scale invariant, corresponds to a flat distribution in the intersection area of the branes, with a maximum area $A_{\max} \sim 100\alpha'$.

1 Introduction

It is an empirical fact, yet a great puzzle, that the universe began out of equilibrium. While we expect that every theory has at least one vacuum state, the universe did not make use of such a state. We remain out of equilibrium even today. Present indications suggest that we are approaching a state consisting of de Sitter space with a vacuum energy of magnitude $\Lambda_0 = 2.7 \times 10^{-59} \text{ TeV}^4$, although even this may not be the final resting state of the universe. In any case, it is clear from observational evidence that the Universe is not now in a vacuum state, and was not in the past.

Moreover, it is clear that the universe was further from equilibrium in the past. We can reliably trace back the universe to a period of higher temperatures and faster expansion. The ultimate scale describing the departure from equilibrium is less clear. If our 4d world emerged from string theory, it is reasonable to assume that this scale was the mass scale of the 4d effective field theory - E_4 - which we will assume is a fraction of the string scale.

Finally, from observational evidence it appears increasingly likely that scalar field inflation occurred in the early universe. The isotropy of the universe, the overall density with $\Omega = 1$ and the detailed pattern of density fluctuations in the microwave background support this conclusion. This has the implication that our present observable universe emerged from a very small patch of the original universe and remains only a small fraction of the full universe.

These ingredients lead to the expectation that in string theory the full universe should consist of regions which involve different vacuum states. As analyzed in more detail below, the dynamics of transitions between different ground states which occur when the universe is far from equilibrium will lead to different domains with different cosmological constants and other parameters. Rather than having a single ground state that permeates the whole universe, as we tend to assume for most field theories, the lack of equilibrium and the existence of inflation coupled with string theory transitions will lead to a multiple-domain universe.

String theory has very many different vacua which are possible [1, 2]. There are also mechanisms for transitions between them. A simple and well known example of this is the membrane nucleation process initially studied by Brown and Teitelboim [3, 4]. In this situation, bubbles of a new vacuum form in four dimensions with a different value of the effective cosmological constant through the nucleation of a two dimensional membrane coupled to

a four-form field. At present energies, the probability of this nucleation is so small that it is of little cosmological interest. However, in the early universe with inflation there would be regions in which membranes are formed. Moreover, at high energies the bubble formation need not involve tunnelling and I will present estimates for nucleation in a finite temperature state. For a universe that is out of equilibrium by E_4 , these estimates suggest that causally disconnected regions will be related to different ground states. In the subsequent cosmological evolution, inflation would have placed these other regions outside of our observable horizon.

This paper is an exploration of the dynamics and phenomenology of such a universe. The different regions of the present universe would potentially involve different low energy theories or at least different parameters. A serious recent estimate suggests that in there could be very many string vacua (perhaps 10^{100}) that look like that Standard Model with couplings and masses such as are observed, as well as far more with different parameters [1, 2, 5, 6]. This would make it unlikely that the prediction of these parameters would be a test of string theory. I will make a preliminary exploration of a different notion - namely that the quark and lepton masses provide a visible remnant of the early dynamics, reflecting the weight or measure by which the Yukawa couplings are distributed.

2 The distance between vacuum states

The low energy effective field theory is a mapping from string theory parameters (manifolds, moduli, fluxes) to Standard Model parameters (masses and coupling constants). The number of choices for compact manifolds, for embedded fluxes and for branes wrapped on cycles is extremely large. Because of this large number, the possible output parameters of the low energy theory are also quite large and quite possibly are densely packed. Douglas[1] has initiated a program of counting the numbers of string theory vacua. Each vacuum state is a delta function in string theory parameter space, since the fluxes and hence moduli are quantized[7, 8]¹. However if the resulting moduli potentials are closely packed, it makes sense to define an approximate

¹This neglects the possibility of unquantized fluxes that can occur in warped universes with infinite dimensions[9], in which case the number of vacua is infinite and continuous.

measure for the vacuum states

$$d\mu(V) = \sum_{T \in \text{theories}} \delta(V - V(T)) \quad (1)$$

The low energy vacua can appear almost as a continuum if the resulting parameters are dense. The estimates of the total number of possible flux vacua is difficult to estimate precisely but could be of order $\sim 10^{300}$ [1]. Restrictions to those parameters that match the Standard Model was estimated to reduce the number by a factor of $\sim 10^{-140}$. However, this leaves the possibility that there could be even order 10^{100} vacua that reproduce the parameters of the Standard Model *within their present experimental error bars*. However, even if this high degeneracy is not realized and our set of Standard Model parameters is somewhat unique, in the sense that there is only a single string vacua that produces these parameters within the present experimental error bars, the important result is that there are an extremely large number of related vacua that differ by the change of some flux or brane wrapping.

An essential point for the dynamics of these vacua is that two vacua that are close together in Standard Model parameter space are far apart in the parameters of string theory. Likewise the states that are close in string theory parameter space are relatively far apart in their Standard Model parameters. The density of possible Standard Model parameters occurs only because of a great multiplicity of different fluxes vacuum choices. This feature is displayed in the work of Bousso and Polchinski[8]. For the compactification of M theory on a seven-manifold with several nontrivial three cycles, the quantization rules yields a vacuum energy

$$\Lambda = \Lambda_{\text{bare}} + \sum_i n_i^2 \frac{\pi M_{11}^3 V_{3,i}^2}{V_7} \quad (2)$$

where n_i are integers and the V_i are the invariant volumes. The spacing of the allowed values of Λ can be very small even if the scale of each contribution is itself large. Bousso and Polchinski [8] show that if there were of order 100 such fields, the density of states would be so high that the differences between two values of the cosmological constant would be of the order of the experimental value. However, a change of a single flux by one unit would then change the vacuum energy by a factor

$$\Delta\Lambda = \frac{2\pi n_i M_{11}^3 V_{3,i}^2}{V_7} \quad (3)$$

Unless one of the three cycles is exceptionally small, this is a large jump of order the compactification scales. In order to move between the two small values for the cosmological constant that are each of the same size as the experimental value, one would have to rearrange of order 100 different four-form flux values.

Changes in the internal fluxes also lead to changes in the moduli. Generically, non-zero fluxes contribute to the potentials for the moduli fields

$$V \sim \sum_i F_i^2 e^{-a_i \phi} \quad (4)$$

(for example, see Ref. [10] for a discussion of these potentials in a cosmological context.). A transition in the fluxes will readjust the minimum of the moduli potentials leading to changes in the parameters of the low energy Standard Model. The change of one flux by one unit will then also modify the moduli by a significant amount.

This behavior has a consequence for the dynamics of these fields in cosmology. Simple changes in the string theory fluxes do not move between closely related Standard Model vacua with small changes in the parameters. Instead the most likely changes are far apart in the parameters of the Standard Model.

In the work of Feng et. al. [11], an attempt was made to construct string vacua where the cosmological constant had small steps - of the order of the experimental value - when a four form flux was changed by one unit. This would allow the relaxation of the cosmological constant via membrane nucleation, as described in the next section, to proceed by small steps with an end result that could naturally be of order the observed value. However this was accomplished only by going to extremes of string theory parameter spaces, i.e. cycles of vanishing size or tiny string couplings. A more generic vacuum state has larger steps. Moreover, even if there are degenerating cycles, there can be other cycles of normal size in the internal manifold that generate flux potentials. Then there will be additional flux changes that are possible that lead to large jumps in parameter space. In this class of theories there would then be both large and small jumps in the low energy parameters upon membrane nucleation.

3 Dynamical transitions

In an evolving universe there can be transitions between different vacuum states. We will consider two types of transitions. In de Sitter phase, the membrane nucleation of Brown and Teitelboim will be relevant. However, in other circumstances finite temperature nucleation will be relevant. These have not been discussed previously in the literature and we will discuss these in more detail.

3.1 de Sitter transitions

Brown and Teitelboim calculated the probability that a 2d membrane will be nucleated in 4d de Sitter spacetime, with the interior region having a different value of a four-form field strength. The Brown-Teitelboim results are simply presented in the notation of Ref [11] which summarize the formulas for the transitions more compactly than those of the original work.

Four-form field strengths describe fields for whom the local equation of motion require that they be constants in four dimensions. The gauge potential gauge potential is a three-form with a four-form field strength tensor

$$F_{\alpha\beta\gamma\delta} = \partial_{[\alpha} A_{\beta\gamma\delta]} \quad (5)$$

where the square brackets denote the antisymmetrization of the indices. The action

$$S_F = -\frac{1}{48} \int d^4x \sqrt{-g} F_{\alpha\beta\gamma\delta} F^{\alpha\beta\gamma\delta} \quad (6)$$

leads to the equation of motion

$$\partial^\alpha [\sqrt{g} F_{\alpha\beta\gamma\delta}] = 0. \quad (7)$$

The only solution to this is

$$F_{\alpha\beta\gamma\delta} = \frac{c}{\sqrt{g}} \epsilon_{\alpha\beta\gamma\delta} \quad (8)$$

for arbitrary constant c . Thus this field is nondynamical, with only a constant solution. Substitution of this solution in Einstein's equations shows that it behaves as a positive cosmological constant.

Four-form fields are ubiquitous in string theory. Each four-form couples to a 2-brane source, generically labeled B^μ . The action for the coupling of the 2-brane to the gauge potential $A_{\alpha\beta\gamma}$ is

$$S = \int d^3x \left[\tau_0 \sqrt{\det(g_{\alpha\beta} \partial_a B^\alpha \partial_b B^\beta)} + \frac{\rho_0}{6} \epsilon^{abc} \partial_a B^\alpha \partial_b B^\beta \partial_c B^\gamma A_{\alpha\beta\gamma} \right] \quad (9)$$

Here ρ_0 is the charge per unit area of the 2-brane and τ_0 is the tension of the 2-brane. For BPS 2-branes the tension and the charge are related by

$$\tau_0 = \frac{\rho_0 M_P}{\sqrt{2}} \quad (10)$$

In the process of membrane creation, the four-form potential will differ across the membrane. Let us define the interior and exterior values of the four-form by

$$F_{i,e}^{\alpha\beta\gamma\delta} = \frac{c_{i,e}}{\sqrt{g}} \epsilon^{\alpha\beta\gamma\delta} \quad (11)$$

The overall cosmological constant will then have interior and exterior values

$$\Lambda_{i,e} = \Lambda_0 + \frac{1}{2} c_{i,e}^2 \quad (12)$$

Here Λ_0 is the cosmological constant from all other sources. We will use the notation

$$\lambda = 8\pi G \Lambda \quad (13)$$

to differentiate the cosmological constant from the vacuum energy density. Note that the former has units E^2 while the latter has units E^4 .

When there is a transition to a lower value of the cosmological constant, BT show that the bubble size that minimizes the instanton action is equivalent to the condition of energy conservation in membrane formation. Specifically, for de Sitter metrics of the form

$$ds^2 = -(1 - \frac{1}{3} \lambda r^2) dt^2 + (1 - \frac{1}{3} \lambda r^2)^{-1} dr^2 + r^2 d\Omega \quad (14)$$

the energy increase associated with bubble nucleation includes both the vacuum energies and the mass of the membrane,

$$\Delta E(r) = \frac{4}{3} \pi (\Lambda_{int} - \Lambda_{ext}) r^3 + 2\pi \tau r^2 \left[\sqrt{1 - \frac{8\pi G \Lambda_{int} r^2}{3}} + \sqrt{1 - \frac{8\pi G \Lambda_{ext} r^2}{3}} \right] \quad (15)$$

The condition $\Delta E(r = b) = 0$ fixes the radius of the membrane at nucleation

$$b = \left[\frac{9\tau^2}{(6\pi G\tau^2)^2} + 12\pi G\tau^2(\Lambda_{ext} + \Lambda_{int}) + (\Lambda_{ext} - \Lambda_{int})^2 \right]^{\frac{1}{2}} \quad (16)$$

Although this derivation only holds for the case $\Lambda_{int} < \Lambda_{ext}$, the same result holds for the situation where the cosmological constant increases in the interior of the bubble, $\Lambda_{int} > \Lambda_{ext}$.

The rate of nucleation per unit volume has the form

$$\frac{\Gamma}{V} \sim e^{-B} \quad (17)$$

where B is the instanton action

$$B = \frac{3M_P^2}{16} \left[\frac{1}{\Lambda_{ext}}(1 + \cos \theta_{ext}) - \frac{1}{\Lambda_{int}}(1 - \cos \theta_{int}) \right] \quad (18)$$

where the angle factors are given by

$$\begin{aligned} \cos \theta_{int} &= \frac{\Lambda_{ext} - \Lambda_{int} + 6\pi G\tau^2}{[\Lambda_{ext} - \Lambda_{int} + 6\pi G\tau^2]^2 + 24\pi G\Lambda_{int}} \\ \cos \theta_{ext} &= \frac{\Lambda_{int} - \Lambda_{ext} + 6\pi G\tau^2}{[\Lambda_{int} - \Lambda_{ext} + 6\pi G\tau^2]^2 + 24\pi G\Lambda_{ext}} \end{aligned} \quad (19)$$

Despite the overall factor of the Planck mass in B , the rate is independent of the Planck mass when the other scales are much smaller. In the limit, $G \rightarrow 0$ one has

$$\begin{aligned} B &= \frac{27\pi^2}{2} \frac{\tau^4}{(\Lambda_{ext} - \Lambda_{int})^3} \quad (\Lambda_{int} < \Lambda_{ext}) \\ B &= \infty \quad (\Lambda_{int} > \Lambda_{ext}) \end{aligned} \quad (20)$$

This indicates that transitions which decrease Λ can take place without the mediation of gravity while those that increase the cosmological constant only occur as a gravitational effect. A few other limits are also worth displaying. In the limit of small τ , i.e. $6\pi G\tau^2 \ll |\Lambda_{ext} - \Lambda_{int}|$, one has the related limit

$$\begin{aligned} B &= \frac{27\pi^2}{2} \frac{\tau^4}{(\Lambda_{ext} - \Lambda_{int})^3} \quad (\Lambda_{int} < \Lambda_{ext}) \\ B &= \frac{3}{8} \left[\frac{M_P^4}{\Lambda_{ext}} - \frac{M_P^4}{\Lambda_{int}} \right] \quad (\Lambda_{int} > \Lambda_{ext}) \end{aligned} \quad (21)$$

In the limit of small Λ_{ext} , ($\Lambda_{ext} \ll 6\pi G\tau^2$) one finds

$$B = \frac{3}{8} \frac{M_P^4}{\Lambda_{ext}} \quad (22)$$

independent of Λ_{int} , while if Λ_{int} is small ($\Lambda_{int} \ll 6\pi G\tau^2, \Lambda_{ext}$) one has

$$B = \frac{3M_P^2}{8} \frac{((6\pi G\tau^2)^2)}{\Lambda_{ext}(\Lambda_{ext} + 6\pi G\tau^2)^2} \quad (23)$$

3.2 Finite temperature transitions

At finite temperature, transitions need not take place by tunnelling, but can occur through thermal excitation over the barrier². The probability for such transitions is given by the Boltzmann factor to reach the peak of the energy barrier, namely

$$P \sim e^{-\beta E_*} \quad (24)$$

where $\beta = 1/kT$ and E_* is the height of the barrier[12, 13, 14, 15, 16, 17].

In the energy equation, Eq 15, we may safely specialize to the case where $\lambda_{ext}r^2 = 8\pi G\Lambda_{ext}r^2$ is small compared to unity. This is required to be small if the universe is to undergo finite temperature evolution - otherwise it will quickly turn into de Sitter evolution. It is only if the exterior cosmological constant is small that the finite temperature evolution will occur. However, we should allow the possibility that the interior cosmological constant is not small. Thus we will use

$$E(r) = \frac{4}{3}\pi(\Lambda_{int} - \Lambda_{ext})r^3 + 2\pi\tau r^2 \left[1 + \sqrt{1 - \frac{8\pi G\Lambda_{int}r^2}{3}} \right] \quad (25)$$

The radius that corresponds to the peak of the energy barrier is obtained from the condition

$$\frac{dE}{dr} = 0 \quad (26)$$

First consider transitions where the value of the cosmological constant decreases. Since we already have $G\Lambda_{ext}$ small, we also will have $G\Lambda_{int}$ small.

²If the membrane instanton is the analogue of the Schwinger mechanism producing an e^+e^- pair in a background electric field, the thermal excitation discussed in this section is the analogue of a thermal fluctuation producing an electron positron pair.

The energy is maximum at a radius

$$r_* = \frac{2\tau}{\Lambda_{ext} - \Lambda_{int}} \quad (27)$$

and will have the value

$$E_* = \frac{16\pi}{3} \frac{\tau^3}{(\Lambda_{ext} - \Lambda_{int})^2} \quad (28)$$

Note that this greatly favors transitions which lead to large changes in the cosmological constant.

Because of the gravitational expansion, one can also have transitions to a higher value of the cosmological constant. These occur through a thermal fluctuation which is large enough that the interior expands. The tension would make bubble contract. However if the interior is large enough the expansion of the interior can win. In analyzing this situation, let us consider the case where $\Lambda_{int} \gg \Lambda_{ext}$. The energy function can then be rescaled using the variable

$$x^2 = \frac{8\pi G \Lambda_{int}}{3} r^2 \quad (29)$$

$$\begin{aligned} E(r) &= \frac{4}{3} \pi \Lambda_{int} r^3 + 2\pi \tau r^2 \left[1 + \sqrt{1 - \frac{8\pi G \Lambda_{int} r^2}{3}} \right] \\ &= \frac{3\tau}{4G\Lambda_{int}} \left[\eta x^3 + x^2(1 + \sqrt{1 - x^2}) \right] \end{aligned} \quad (30)$$

where

$$\eta = \sqrt{\frac{\Lambda_{int}}{6\pi G \tau^2}} \quad (31)$$

The equation for the maximum of $E(r)$ is

$$\frac{dE}{dx} = 0 = 3\eta x + 2(1 + \sqrt{1 - x^2}) - \frac{x^2}{\sqrt{1 - x^2}} \quad (32)$$

Since η is positive, this will have a solution only due to the third term, which can be traced back to the de Sitter metric factor in the energy. Hence we know that this solution only exists due to the action of gravity. The

interesting limiting cases correspond to energies at maximum of

$$\begin{aligned}
E_* &= \frac{8\tau}{9G\Lambda_{int}} & \eta \ll 1 \\
&= \sqrt{\frac{3}{32\pi G^3\Lambda_{int}}} & \eta \text{ large}
\end{aligned} \tag{33}$$

It is not hard to solve for the energy numerically at any given value of η , but the form

$$E_* = \frac{8\tau}{9G\Lambda_{int}} \left[1 + \frac{9}{8} \sqrt{\frac{3\Lambda_{int}}{32\pi G\tau^2}} \right] \tag{34}$$

provides an excellent interpolating formula for intermediate values of η . In each case, transitions to the largest values of Λ_{int} are favored.

4 Multi-domain eternal inflation

Let us follow the logic dictated by the assumptions described above. In this section, we will systematically explore the possible evolutionary pathways of an energetic early universe. The major ingredients are the assumption that scalar-field inflation has taken place in the past evolution of our domain and that the initial state of the universe was at an energy scale, called E_4 below, which was greater than the scale associated with scalar-field inflation.

4.0.1 Types of eternal inflation

Essentially all of the pathways will involve at least one inflationary epoch. There are two types of inflationary expansion that are worth distinguishing here 1) Inflation driven by the energy density associated with a scalar field [18] and 2) Inflation dominated by the energy associated with a four-form field. In reality both the energy density of the scalars and the four-forms contribute to the single parameter - the cosmological constant - that drives the de Sitter phase [19]. However, the important distinction comes in how the inflationary phase ends or changes. With a scalar field, the field eventually will roll down a potential and dump all of its energy which is converted into particles in the process of reheating. This leads to the inflationary prediction that $\Omega = 1$. On the other hand a four-form makes a transition to a different value through the process of membrane nucleation. The membrane will carry off some of the energy as the bubble expands, leaving the interior region with

less than the critical density, $\Omega < 1$. For this reason, four-form inflation is not a good candidate for the final inflationary phase which appears to have taken place in our portion of the universe [20].

It is important to keep in mind that both forms of inflation are generically future eternal. In the case of scalar field inflation this is well known[21, 22, 23, 24]. The scalar field that is responsible for inflation will in some place fluctuate higher up the potential - in others it will fluctuate down. While inflation will end in some regions as the field moves down the potential, there will always be other domains that continue to inflate. In the case of four-form inflation the reasoning is different. The membrane nucleation process that changes the cosmological constant only does so on the interior of a finite region. The exterior region continues to inflate at the old value of the cosmological constant. Because the bubble expands at the speed of light while the de Sitter expansion is more rapid, the bubbles do not fill the space and there are always the exterior regions that are still inflating³. Thus once the universe enters an inflationary phase, it will always be inflationary except in domains in which transitions to small values of the cosmological constant has taken place.

4.1 deSitter dominated

We have assumed that the initial state of the universe is out of equilibrium by an amount labelled E_4 . There will be various evolutionary pathways possible. An immediate distinction is whether this energy is manifest as a cosmological constant of order $\lambda = G\Lambda \sim GE_4^4$ or if it appears in the form of energetic particles. In the former case the universe will immediately be in a de Sitter phase (a “cold” start), while in the latter we will argue that it thermalizes (a “hot” start). The pathways will then branch out from these cases depending on the nature of the subsequent transitions.

4.1.1 From deSitter to de Sitter

If vacuum energy is of order $\Lambda \sim E_4^4$, then the cosmological constant is

$$\lambda = 8\pi G\Lambda \sim \frac{E_4^4}{M_P^2} \quad (35)$$

³This pattern also occurred in “old inflation” in which there was a first order phase transition.

The initial state is then one of de Sitter expansion with the dominant ingredient being the cosmological constant from the four-form field.

In a de Sitter phase, the Brown-Teitelboim nucleation will be operational. Bubbles of different values of the cosmological constant will be formed in the overall de Sitter phase. Because the inflation is future eternal, there will be an ever increasing number of the bubbles formed even if the probability for any one bubble is not large. These transitions can be either to smaller or to larger values of the cosmological constant. While transitions to larger values are possible in the Brown-Teitelboim calculation, they may not be physically possible if they correspond to such a large value that the low energy effective theory is no longer applicable. However, if the effective theory is possible, then inflation simply continues and further membrane nucleation takes place.

The regions that are finally most interesting to us are those that make a transition to a smaller value of the cosmological constant. In these regions other behaviors are possible. In some regions, there will be a transition to a lower cosmological constant. Let us first neglect the possibility of particle production during this transition. In this case, the patch of the universe will transition to another de Sitter region. If the cosmological constant in this region is dominated by the four-form field, we will repeat the above set of choices. If it is dominated by the scalar field, we enter a period of scalar field inflation. The subsequent evolution of such a domain is standard for the inflationary literature. It involves an inflationary period, a brief roll-down for the scalar field and a reheating period, followed by thermal evolution of the universe. Some of these domains have the potential to evolve into the universe that we see observationally.

4.1.2 From deSitter to thermal to inflation

Another possibility is that at the time of the Brown-Teitelboim transition, the domain experiences significant particle production. This is not present in the original Brown-Teitelboim calculation, but it would clearly be present in string theory. This is because the form fields also contribute to the moduli potentials. A change in the moduli potentials will lead to a modification of the masses and couplings of the low energy theory. Anytime these parameters change abruptly there will be particle production. For example, the moduli potential will experience a shift of order

$$\Delta V \sim \Delta F^2 e^{-a\phi} \quad (36)$$

Since this is a steep potential the field will rapidly seek its new minimum, producing particles as it readjusts. The reheating temperature cannot be calculated without knowledge of the other contributions to the moduli potential but, since it is scaled by the change in the vacuum energy with no extremely small parameters involved, the reheating temperature should be a fraction of the original vacuum energy. These domains will resemble an open universe with a density below critical density.

In the presence of particle production, there will be some domains that will evolve differently from the above description. In these domains the residual cosmological constant is smaller than the reheating temperature. These will then evolve as a radiation dominated universe rather than as a de Sitter state. However, since the last transition is that of four-form inflation, these domains have a density less than the critical density and they will evolve to an almost empty open universe unless de Sitter expansion takes over again in their future. Such domains are not like ours. We could not find ourselves in an empty domain so that we need not consider this case further.

4.2 Particle dominated initial conditions

In this section, we consider the case where the cosmological constant is smaller than other forms of energy contained in the fields. In the absence of the expansion of the universe it is clear that any such energetic initial condition would eventually lead to thermal equilibrium. In the presence of expansion one must ask if the rate of expansion prevents the system from reaching an approximate equilibrium[25]. Individual cross sections and particle densities at an energy E will be of order

$$\sigma \sim \frac{\alpha^2}{E^2} \quad n \sim E^3 \quad (37)$$

where α is the coupling strength of the interactions. At high energies, the gauge interactions are characterized by an interaction strength of order $\alpha \sim 1/25$. Reactions that lead to thermalization then occur at a rate

$$\text{rate} \sim g_* \sigma n \sim g_* \alpha^2 E \quad (38)$$

where g_* describes the number of particles available, which could be of order 10^2 . On the other hand, gravitational expansion could prevent equilibrium

from occurring if it is more rapid than the equilibration rate. The expansion rate is

$$H \sim \sqrt{G\rho} \sim \sqrt{g_* G E^4} \quad (39)$$

The ratio of these rates is

$$\frac{\sigma n}{H} \sim \frac{g_*^{1/2} \alpha^2}{\sqrt{G E^2}} \quad (40)$$

We see that unless the energy scale is close to the Planck scale, the universe will thermalize. In the case that the universe starts out close to the Planck scale, there will not initially be thermal equilibrium. However the expansion will scale down the energy by a factor of the scale factor

$$E(t) \sim \frac{a(t_0)}{a(t)} E_0 \quad (41)$$

In this way the energy density will eventually fall to a value which does allow for thermal equilibrium. Thus in cases where the cosmological constant does not play a role in gravitational expansion, the gauge interactions will eventually lead to thermal equilibrium.

In this situation, finite temperature transitions are initially important. As described in Sec 3, this can result in thermal creation of regions of different values of the cosmological constant. Transitions with a large change of the cosmological constant are favored. Initially, the effect of the cosmological constant is subdominant - it is hidden beneath the larger thermal energy. The subsequent evolution of the various domains depends on the magnitude of the cosmological constant generated by the thermal transitions.

4.2.1 From thermal to deSitter

Since the thermal transitions are exponentially suppressed, most regions of the universe will evolve without any change in the value of the cosmological constant. However, as the thermal region cools it will generically enter a period of inflation. (The special case of no inflation is dealt with below.) Depending on the magnitude of the different contributions to the cosmological constant, this can be either scalar field inflation or four-form inflation. In either case, the description of the universe becomes that of de Sitter expansion and the energy contained in the field degrees of freedom rapidly goes to zero. If the four-form fields dominate the cosmological constant, the situation reverts to the analysis of the previous subsection. If it is the scalar field energy

that dominates the cosmological constant, then the usual scenario of eternal scalar field inflation results. However, in addition to the fluctuations of the scalar field, one needs to account for the Brown-Teitelboim fluctuations of the four-form field, also described above. Hence in either of these cases, we arrive at a universe of eternal inflation with domains that have fluctuated to different values of the cosmological constant. A subset of these domains can appear similar to our own.

In some regions, transitions to other values of the cosmological constant will occur. If the resulting cosmological constant remains smaller than the thermal energy density, the analysis of the previous paragraph remains valid. If the final cosmological constant is larger than the local thermal energy, then the universe is locally de Sitter dominated. The subsequent history of these domains then follows the pathways described above.

Finally, there will be regions that settle into a low energy state (with energy $E < E(\text{inflation})$) without going through any form of inflation. For this to occur, the total value of the cosmological constant needs to be small, so that both the four-form contribution and the scalar field contribution are individually small. Such regions can exist if the initial cosmological constant is tiny or if there was a thermal fluctuation to a tiny value of the cosmological constant. These regions are excluded from being our observable universe by the initial assumption that our domain has gone through a period of scalar field inflation. Moreover, a region such as this is an infinitesimal fraction of the allowed regions as long as a single transition occurs to start the process of eternal inflation.

4.3 All pathways lead to inflating domains

Overall, this analysis has lead us to a universe that has regions of eternal inflation with different values of the expansion rate. In this sense, such a multi-domain inflating universe is an attractor solution. For all energetic initial states the dynamical transitions which are possible in string theory will lead to this as the final state. The different regions will span the various allowed values of the physical parameters. In a subset of those regions, inflation will have ended and a matter dominated era can occur. Some of these regions will be similar to our own.

5 Typicality and the strong CP problem

Even if there are very many domains, this does not absolve us from needing to understand the structure of fundamental theory in our domain. Physics is an experimental science and we experimentally explore the nature of our domain. However, the multiple domain structure does change the nature of some of the key questions. Within the framework under discussion I will argue that this selects the strong CP problem as a more important problem than the hierarchy problem or the cosmological constant problem⁴.

There are three fine-tuning problems that are usually highlighted as violations of the principle of “naturalness” - the cosmological constant problem, the hierarchy problem and the strong CP problem. The principle of naturalness states that it is unnatural for a parameter in a theory to be much smaller than the magnitude of the radiative corrections to that parameter. If a parameter is unnaturally small, then there must be a fine-tuned adjustment of many large contributions to sum up to a value much smaller than any of the individual contributions. Since this is aesthetically distasteful, it motivates searches for new dynamical mechanisms in which the smallness of the parameter is natural. The expectation that we will find new dynamics at around 1 TeV is primarily motivated by the argument of naturalness for the Higgs vacuum expectation value. Note that naturalness is a somewhat fuzzy concept - witness the discussions of whether the present constraints on supersymmetry make that theory unnatural. However, it is an effective motivator for deciding which problems are important to study for indications of new physics.

In a theory with multiple domains, naturalness is not as useful a concept. In the ensemble of domains there will always be some with unnaturally small values for the parameters. In addition, whether we like it or not, one must unavoidably take into account “anthropic” boundary conditions [26]. Such constraints will sometimes select only those domains which are technically unnatural. For example, we could not conceivably find ourselves in a domain with natural values of the cosmological constant, so we inevitably must constrain our consideration to domains with a viable value of the cosmological constant.

In multiple domain theories, there is a different concept that can replace

⁴While this paper was being written up, Ref. [6] appeared which, containing related comments on the strong CP problem and supersymmetry breaking.

naturalness. Within a given fundamental theory with a particular history, the ensemble of domains will define a distribution for the parameters of the low energy theory. Some of these parameters may be restricted if we specialize to the anthropically allowed domains, yet others will still have a significant range and distribution. We would expect that our domain should be typical of this ensemble, subject to anthropic constraints. The parameters that we find should not be extremely unusual for the ensemble of viable domains. I will refer to this expectation as “typicality”⁵ and will provide examples of how it changes the motivation for new dynamics beyond the standard model in multiple domain theories. In the next section, I will extend this notion to obtain specific information on the initial ensemble of domains.

There are only a few parameters of the Standard Model which have significant anthropic constraints. Primary among these are the cosmological constant and the Higgs vacuum expectation value. Physically, these constraints are manifest in the requirement that the universe allows matter to clump into stars- which strongly restricts the values of the cosmological constant[27] - and in the need for atomic elements beyond hydrogen to be stable - which only occurs for a narrow range of the Higgs vev[28]. In a multiple domain realization of string theory, we would restrict our attention to only those vacua that satisfy these constraints. Thus the naturalness or fine-tuning problems of these two parameters are not significant problems for such multiple domain theories.

However, the strong CP problem appears to be in severe conflict with typicality. In the Standard Model the θ parameter, which measures the amount of strong CP violation, is a dimensionless coupling constant. It is infinitely renormalized by radiative corrections and there is no reason within the theory for it to be small. There are also no known anthropic constraints on the value of θ - the world would be essentially unchanged for θ of order unity. Nevertheless, experimental bounds on the electric dipole moment of the neutron constrains

$$\theta \leq 10^{-10} \quad . \quad (42)$$

⁵Vilenkin [29] has provide a more specific formulation of this idea under the name of the “principle of mediocrity”. Vilenkin’s principle of mediocrity has a technical difference with my usage of typicality. The former is defined by a measure which is proportional to the number of civilizations in a given domain. While this emphasis on the number of civilizations may be laudable in certain contexts, I prefer not to include it in the present discussion. Moreover, mediocrity has the unfortunate connotation of “not good enough”, while we have a pretty damn good universe.

This is a potential problem for multiple domain theories. A priori, one would expect that the ensemble of viable Standard Model domains would have a distribution of θ that would span values much larger than the experimental constraint by up to ten orders of magnitude. If this is the case, we would have to conclude that we are a very non-typical domain. Since it is not very likely that we would randomly find ourselves in such a domain, we need to seek dynamical solutions to the strong CP problem. Moreover it suggests that these solutions must be generic in string theory - that they occur in a typical vacuum solution. This is a strong constraint. In multiple domain theories, the value of θ appears more puzzling than that of Λ or v .

Typicality may also have other implications. As mentioned above, a small cosmological constant and a low Higgs vev are required by anthropic constraints, so we should only look for vacua that live within this range and ask if our vacuum is typical of this range. But this does not exhaust the issue of typicality. For example, in principle one could decide if low energy SUSY is likely in string theory. One would do this by counting the number of available string vacua. Are there more viable SM vacua with low energy SUSY breaking or with high energy breaking? These numbers are unlikely to be similar. As a hypothetical extreme example, consider a situation where there are of order 10 viable SM vacua with Planck scale SUSY breaking and 10^{120} with weak scale breaking. The differences in these numbers could arise because of the need to have an appropriately small cosmological constant, which might be less likely if the SUSY scale is larger. In this case, typicality predicts that we should find supersymmetry at low energy. This would be a *statistical* prediction, but could be compelling - the numbers are potentially just too overwhelming. Note also that this prediction could be different from naturalness - the numbers of the different vacuum states could in principle be reversed such that there are more viable vacua with high energy SUSY breaking.⁶ The counting of these numbers of states is a well-posed question that can in principle be addressed in string theory.

Dark matter is also a potential problem with the principle of typicality. Matter of any form was inessential in early universe, yet in our present universe there are comparable mass densities from ordinary matter and dark matter - modulo a factor of 5. If matter and dark matter come from different

⁶Michael Douglas (private communication) has also suggested that this distinction could be indicative of the mechanism of SUSY breaking - perhaps there are far more vacua with an appropriate Λ with gauge mediated SUSY breaking than gravity mediated breaking because of the lower mass scale associated with the former.

sectors of the fundamental theory and correspond to quite different masses, why should their densities be so close in the universe? It is not enough to find some values of the parameters that allows such a relation - it must be a typical case. The generic solution to the nature of dark matter should therefore likely have both dark matter and ordinary matter tied to the same mechanism of production.

6 Quark masses as a probe of vacuum dynamics

If the parameters of the low energy theory are not unique predictions of string theory, and we have only one domain to observe, it appears difficult to use the parameters as probes of string theory. However, there are many parameters in the Yukawa sector and these may be subjected to a statistical analysis. We have 6 quark masses, 3 lepton masses and the CKM parameters which are representative of the distribution of Yukawa couplings. Even though this is not a very large number of observables, we can still use them to obtain valuable information on the underlying theory.

The basic idea is that if there are enough vacuum states consistent with the Standard Model, the quark masses could appear as random variables distributed with respect to some weight. Even if we held fixed one mass, there could be an ensemble of domains that have a variety of mass values for the other quarks. The masses which are observed would not be uniquely predicted but would be representative of this ensemble. Observationally, the masses are not uniformly distributed - they are most numerous at low mass, yet extend out to the very large top quark mass. As I will discuss below, they are quite close to a *scale invariant* shape $\rho(m) \sim 1/m$. This weighting of the masses would then be the observational remnant of the original ensemble.

In what follows, I will treat the quark masses as independent quantities. I will also neglect any anthropic constraints, aside from some brief comments below. Neutrino masses will not be considered here as they likely involve a different mechanism from the other fermion masses.

The weight or measure of quark masses is defined as follows[30]. In an ensemble of domains similar to our own, with the other Standard Model parameters equal to ours, the fraction of masses found at a value m within

a range dm is defined to be

$$f(m) = \rho(m) dm \quad (43)$$

where $\rho(m)$ is the symbol for the weight. The normalization of the weight is

$$1 = \int \rho(m) dm \quad . \quad (44)$$

The weight depends on the scale at which it is defined, and the renormalization group equations for the weight were worked out in Ref. [30]. The comments below apply to the weight at the weak scale.

One cannot simply plot the observed masses in order to reproduce the shape of $\rho(m)$, because the masses would form a delta function distribution. One needs to smooth this distribution in order to compare theory and experiment. One way to do this involves taking the Hilbert transform

$$H(z) = \int_0^\infty dm \frac{z\rho(m)}{m+z} \quad (45)$$

Here one can use the experimental masses to produce an $H_{\text{expt}}(z)$, and compare that form to the transform of various trial forms for $\rho(m)$. For the experimental side we use

$$\rho_{\text{exp}}(m) = \frac{1}{N} \sum_{i=1}^N \delta(m - m_i) \quad (46)$$

In [30], the uncertainties associated with the experimental distribution were assessed by variously dropping one of the quark masses from consideration (to simulate the limited amount of statistics involved), by adding lepton masses either raw or rescaled by a renormalization group factor, and by various ways of including the CKM matrix elements. The experimental transform and its estimated uncertainty are shown in Fig. 1.

In Ref [30] power-law weights were considered. One form was a pure power behavior combined with a cutoff at the quasi-fixed point of the renormalization group $m^* \sim 220$ GeV

$$\rho_1(m) = \frac{N}{m^\delta} \Theta(m^* - m) \quad (47)$$

Here $\delta < 1$. The best fit was found for $\delta = 0.91$, and the result is shown in Fig. 2. Also shown in this figure are the results for weights with lower powers

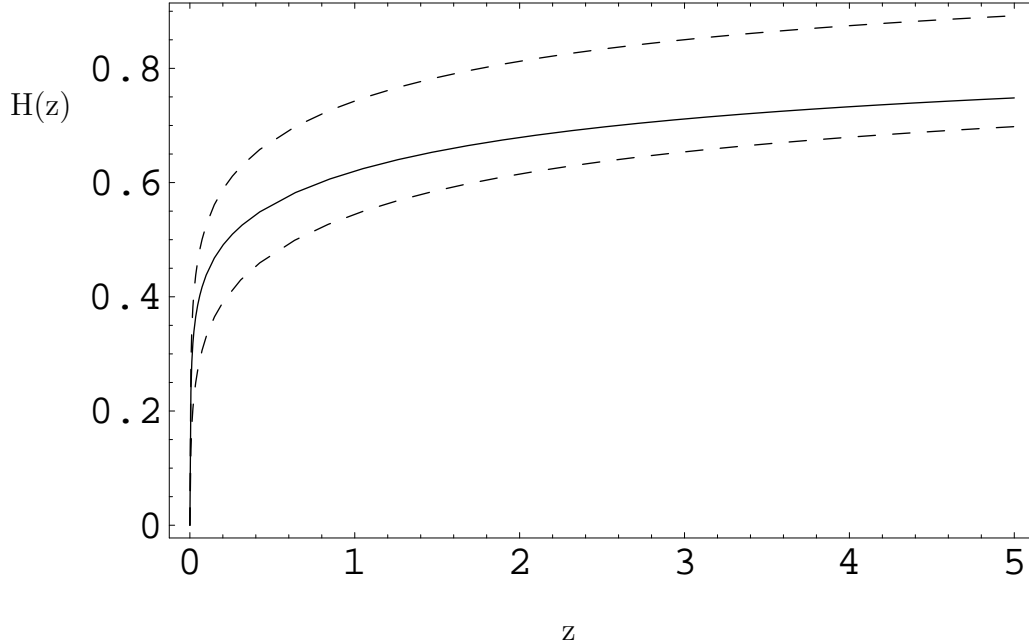


Figure 1: The transformed weight $H(z)$ corresponding to the experimental values of the quark masses defined at the scale M_W (solid curve). The dashed curves are estimates [30] of the upper and lower ends of the allowed range of this quantity due to the limited statistics in the number of quark masses.

of δ . One can see that the experimental range does significantly constrain the form of the weight and that powers near unity are favored. Because of this fact, it is also useful to specifically consider a scale invariant form with $\delta = 1$. Here we require a cutoff at low mass if the distribution is to be normalizable. We can form a normalizable

$$\rho_3(m) = \frac{N}{m} \theta(m - m_{\min}) \theta(m^* - m) \quad (48)$$

with $N = 1/\ln(m^*/m_{\min})$. The results depend only weakly (logarithmically) on the lower cut-off. The result plotted in Fig 3 uses $m_{\min} = 0.1m_e$. A weight with $\rho(m) \sim 1/m$ can be described as “scale invariant” in the following senses. In the first place, there is no scale in the shape of $\rho(m)$, and the normalization constant is dimensionless and independent of the overall scale. In addition, under any linear rescaling of the masses such as occurs for the

scale dependence of QCD,

$$m_2(\mu_2) = \left(\frac{\alpha_s(\mu_2)}{\alpha_s(\mu_1)} \right)^{d_m} m_1(\mu_1) \quad (49)$$

the renormalization group transformation rule[30] tells us that this weight will remain unchanged (again, aside from the endpoints), since

$$\rho_\mu(m) = \rho_{\mu_1} \left(m \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_1)} \right)^{-d_m} \right) \left(\frac{\alpha_s(\mu)}{\alpha_s(m\mu_1)} \right)^{-d_m} \quad (50)$$

$$= \frac{1}{m \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_1)} \right)^{-d_m}} \left(\frac{\alpha_s(\mu)}{\alpha_s(\mu_1)} \right)^{-d_m} \quad (51)$$

$$= \frac{1}{m}. \quad (52)$$

It is intriguing that a scale invariant weight is close to the distribution seen by experiment.

At this point let us specialize to the Intersecting Braneworld models[31, 32, 33, 34, 35], because these have a concrete realization of the physics underlying the Yukawa couplings of the Standard Model. In these theories, chiral fermions live at the intersections of Dp branes in compactified extra dimensions. The right-handed and left-handed fermion fields occur at different intersections and the Higgs field exists at a third intersection. These fields then do not have direct contact interactions, but the Yukawa couplings connecting them through the world sheet instantons with an action proportional to the area connecting the intersections. For a Yukawa coupling h_{ijk} connecting left-handed fermion ψ_{Li} , right-handed fermion ϕ_{Rj} and Higgs H_k one has[31]

$$h_{ijk} = h_0 e^{-\frac{A_{ijk}}{2\pi\alpha'}} e^{2\pi i\phi} \quad (53)$$

where A_{ijk} is the worldsheet area of the triangle connecting the three intersection points of the three branes.

If the intersections relevant for the quark and lepton Yukawa couplings are determined dynamically in the early universe, then the distribution that we see for masses will reflect the distribution of brane intersections. What is interesting is that a very simple distribution results. The scale invariant weight for quark masses described above corresponds to a *flat* distribution of

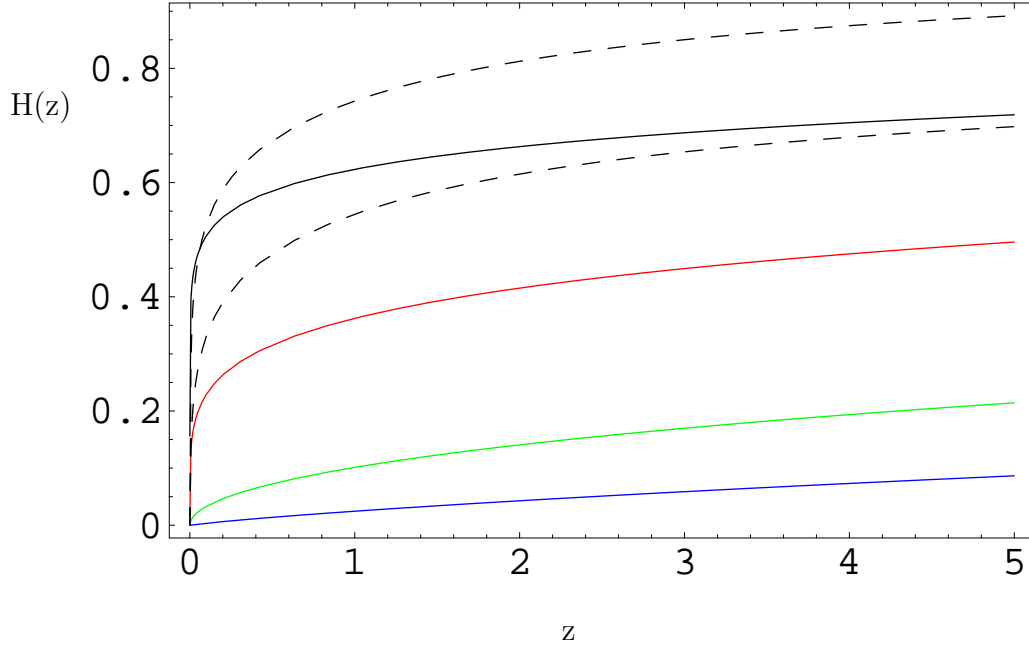


Figure 2: The transformed weights $H(z)$ corresponding to a power law weight with exponent $\delta = 0.91$ (black solid curve) compared to the allowed experimental range (dashed curves). Also shown for comparison below the allowed range are power law weights with exponents $\delta = 0.8$ (red), $\delta = 0.5$ (green) and a flat weight $\delta = 0$ (blue), which shows that these weights are experimentally disfavored.

the worldsheet areas. To see this we note that the exponential dependence in the area implies that

$$\frac{dm}{m} = \frac{dh}{h} = -\frac{dA}{2\pi\alpha'} \quad (54)$$

Therefore if we define a weight or distribution for the areas via

$$\rho(m)dm \sim \rho(A)dA \quad (55)$$

then the scale invariant weight $\rho(m) \sim 1/m$ implies a flat weight for the area

$$\rho(A) = \text{constant} \quad (56)$$

A flat distribution of the areas is perhaps the most natural distribution.

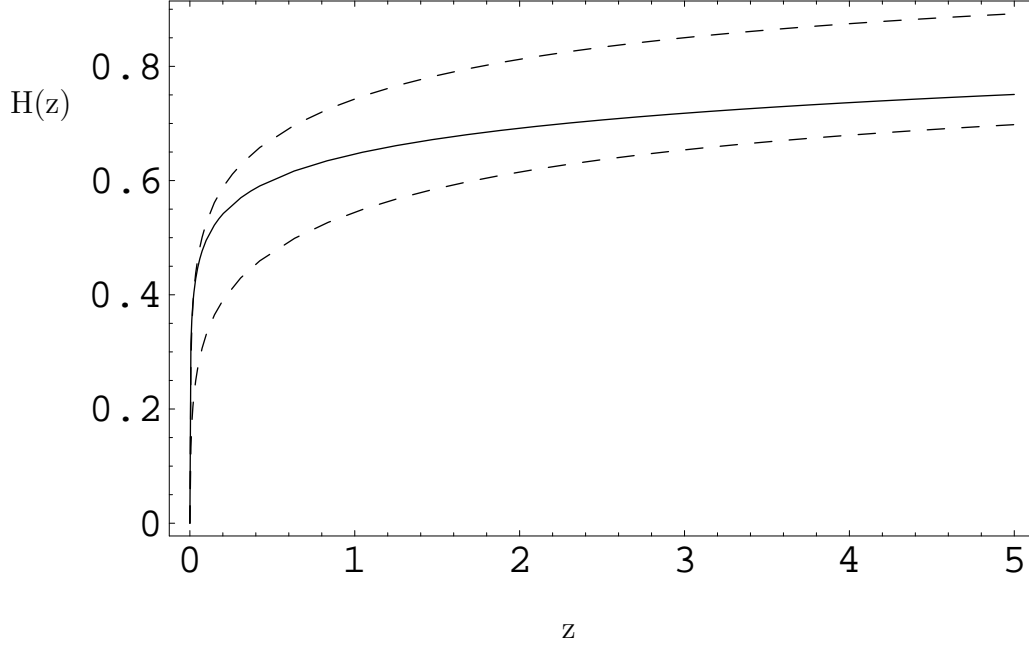


Figure 3: The transformed weight $H(z)$ corresponding to the scale invariant weight $\rho \sim 1/m$ (solid curve), compared to the allowed experimental range (dashed curves)

There is also somewhat weaker information in the limits of the weight function if the shape is exactly the scale invariant form. A flat distribution cannot extend to arbitrarily large areas and still be normalizable. The upper range of the area determines the ratio of the minimum mass to the maximum mass. The largest possible values of the masses are obtained for areas close to zero, which of course is always able to be realized. The minimum mass corresponds to the maximum area, via

$$h_{min} = h e^{-\frac{A_{max}}{2\pi\alpha'}} \quad (57)$$

Thus in the ratio of masses, the overall scale drops out and one finds

$$\frac{m_{min}}{m_{max}} = e^{-\frac{A_{max}}{2\pi\alpha'}} \quad (58)$$

If we take the minimum mass to be $m_{min} \sim 0.1m_e$, we find

$$A_{max} \sim 2\pi\alpha' \ln\left(\frac{m_t}{0.1m_e}\right) \sim 100\alpha' \quad (59)$$

Phenomenologically, there is only a weak constraint on the minimum mass as it enters the weight only logarithmically. However, the minimum mass also only enters logarithmically in the relation for the maximum area.

One of the outcomes of the intersecting braneworld models is then that a uniform distribution in the internal parameters of the model, in this case the area, translates to a very non-uniform distributions in the parameters of the low energy effective theory. It is also interesting that the natural distribution in the model goes a long way towards explaining the very puzzling distribution of fermion masses seen in experiment - with an increasing density at low mass. What remains is to better understand the dynamics that can produce the flat distribution of areas.

In this analysis, we have not considered the impact of anthropic constraints. As discussed in [30] these have the possibility of distorting the experimental weight. Further work would be required to determine the consequences of anthropic constraints on this analysis.

7 Final comments

The initial assumptions are 1) an initial string theory state that is out of equilibrium by an amount $E_4 > E_{\text{inflation}}$, and 2) that our domain underwent scalar-field inflation in the past. The primary physics ingredients have been 1) a huge number of vacuum states as suggested in string theory, 2) the dynamical mechanisms for transitions between them (tunnelling and thermal transitions). In this situation, the universe that results will generically consist of multiple domains related to different vacua.

Linde[18] has emphasized how inflation creates the opportunity for domains with different physical parameters, because an inflating universe leads to a domain structure of regions inflating at different rates. However, this is not sufficient. One also needs a mechanism for producing different couplings in the different regions - otherwise all domains will eventually settle down to the same vacuum state. String theory has such physical mechanisms. The transitions between different flux vacua -whether in de Sitter expansion or at finite temperature - will necessarily produce differing parameters in some domains in an large and eternally inflating universe.

Under the assumptions of this paper, the physics of our domain is seen to be determined not by a condition selecting a unique ground state from string theory, but from the particular past history of our patch of the universe.

The attractor solution that we have found consists of eternal inflation, with various domains in which inflation has ended. Our domain is presumably one of these domains and is clearly one which has satisfied certain anthropic boundary conditions. In such a universe, it is unfortunately difficult to make unique predictions. However, we have raised at least the hope of testing this picture through the use of statistical predictions such as solution of various of the typicality problems, such as the strong CP problem, and through the spectrum of fermion masses.

Acknowledgements

I would like to thank Michael Douglas, David Kastor, Jussi Kalkkinen and Thibault Damour for useful discussions, and to thank the IHÉS for hospitality during much of this work. This work has been partially supported by grants from the National Science Foundation and the Templeton Foundation.

References

- [1] M. R. Douglas, “The statistics of string / M theory vacua,” JHEP **0305**, 046 (2003) [arXiv:hep-th/0303194].
- [2] S. Ashok and M. R. Douglas, “Counting flux vacua,” arXiv:hep-th/0307049.
- [3] J. D. Brown and C. Teitelboim, “Neutralization Of The Cosmological Constant By Membrane Creation,” Nucl. Phys. B **297**, 787 (1988).
- [4] J. D. Brown and C. Teitelboim, “Dynamical Neutralization Of The Cosmological Constant,” Phys. Lett. B **195** (1987) 177.
- [5] L. Susskind, “The anthropic landscape of string theory,” arXiv:hep-th/0302219.
- [6] T. Banks, M. Dine and E. Gorbatov, “Is there a string theory landscape?,” arXiv:hep-th/0309170.
- [7] J. Polchinski, *String Theory* (Cambridge University Press, Cambridge, 1998)

- [8] R. Bousso and J. Polchinski, “Quantization of four-form fluxes and dynamical neutralization of the cosmological constant,” JHEP **0006**, 006 (2000) [arXiv:hep-th/0004134].
- [9] R. Kallosh and A. Linde, “M-theory, cosmological constant and anthropic principle,” Phys. Rev. D **67**, 023510 (2003) [arXiv:hep-th/0208157].
- [10] A. Lukas, B. A. Ovrut and D. Waldram, “Stabilizing dilaton and moduli vacua in string and M-theory cosmology,” Nucl. Phys. B **509**, 169 (1998) [arXiv:hep-th/9611204].
- [11] J. L. Feng, J. March-Russell, S. Sethi and F. Wilczek, “Saltatory relaxation of the cosmological constant,” Nucl. Phys. B **602**, 307 (2001) [arXiv:hep-th/0005276].
- [12] J. Langer, Ann. Phys. (NY) **41** 108 (1967); **54** 258 (1969).
- [13] A. D. Linde, “Fate Of The False Vacuum At Finite Temperature: Theory And Applications,” Phys. Lett. B **100**, 37 (1981).
- [14] A. D. Linde, “Decay Of The False Vacuum At Finite Temperature,” Nucl. Phys. B **216**, 421 (1983) [Erratum-ibid. B **223**, 544 (1983)].
- [15] I. Affleck, “Quantum Statistical Metastability,” Phys. Rev. Lett. **46**, 388 (1981).
- [16] D. J. Gross, R. D. Pisarski and L. G. Yaffe, “QCD And Instantons At Finite Temperature,” Rev. Mod. Phys. **53**, 43 (1981).
- [17] J. Garriga, “Nucleation Rates In Flat And Curved Space,” Phys. Rev. D **49**, 6327 (1994) [arXiv:hep-ph/9308280].
- [18] A. Linde, “Lectures on inflationary cosmology,” hep-th/9410082. A. Linde, *Particle Physics And Inflationary Cosmology*, (Chur, Switzerland, Harwood, 1990).
- [19] S. Kachru, R. Kallosh, A. Linde and S. P. Trivedi, “De Sitter vacua in string theory,” arXiv:hep-th/0301240.
C. Escoda, M. Gomez-Reino and F. Quevedo, “Saltatory de Sitter string vacua,” arXiv:hep-th/0307160.

- S. Kachru, R. Kallosh, A. Linde, J. Maldacena, L. McAllister and S. P. Trivedi, “Towards inflation in string theory,” arXiv:hep-th/0308055.
- M. Gutperle, R. Kallosh and A. Linde, “M / string theory, S-branes and accelerating universe,” JCAP **0307**, 001 (2003) [arXiv:hep-th/0304225].
- C. P. Burgess, R. Kallosh and F. Quevedo, “de Sitter string vacua from supersymmetric D-terms,” arXiv:hep-th/0309187.
- [20] R. Bousso and A. Chamblin, “Open inflation from non-singular instantons: Wrapping the universe with a membrane,” Phys. Rev. D **59**, 063504 (1999) [arXiv:hep-th/9805167].
- [21] A. Vilenkin, “The Birth Of Inflationary Universes,” Phys. Rev. D **27**, 2848 (1983).
- [22] P. J. Steinhardt, “Natural Inflation,” UPR-0198T *Invited talk given at Nuffield Workshop on the Very Early Universe, Cambridge, England, Jun 21 - Jul 9, 1982*
- [23] A. D. Linde, “Eternally Existing Selfreproducing Chaotic Inflationary Universe,” Phys. Lett. B **175**, 395 (1986); “Eternal Chaotic Inflation,” Mod. Phys. Lett. A **1**, 81 (1986)
- [24] A. H. Guth, “Inflation and eternal inflation,” Phys. Rept. **333**, 555 (2000) [arXiv:astro-ph/0002156].
- [25] S. Weinberg, *Gravitation and Cosmology*(Wiley, 1972)
- [26] J. Barrow and F. Tipler, *The Anthropic Cosmological Principle* (Clarendon Press, Oxford, 1986).
C.J. Hogan, “Why the universe is just so,” astro-ph/9909295.
R. N. Cahn, “The eighteen arbitrary parameters of the standard model in your everyday life,” Rev. Mod. Phys. **68**, 951 (1996).
- [27] S. Weinberg, “Theories of the cosmological constant,” astro-ph/9610044.
H. Martel, P.R. Shapiro and S. Weinberg, “Likely Values of the Cosmological Constant,” astro-ph/9701099.
A. Linde, “Inflation And Quantum Cosmology,” *Prepared for Les Rencontres de Physique de la Vallee d’Aoste: Results and Perspectives in Particle Physics, La Thuile, Italy, 26 Feb - 4 Mar 1989.*

- J. Garriga and A. Vilenkin, “On likely values of the cosmological constant,” *Phys. Rev. D* **61**, 083502 (2000) [arXiv:astro-ph/9908115].
- J. Garriga and A. Vilenkin, “Solutions to the cosmological constant problems,” *Phys. Rev. D* **64**, 023517 (2001) [arXiv:hep-th/0011262]
- J. F. Donoghue, “Random values of the cosmological constant”, *JHEP* 0008,022 (2000) , hep-ph/0006088
- T. Banks, M. Dine and L. Motl, “On anthropic solutions of the cosmological constant problem,” *JHEP* **0101**, 031 (2001) [arXiv:hep-th/0007206].
- J. D. Bjorken, “Standard model parameters and the cosmological constant,” *Phys. Rev. D* **64**, 085008 (2001) [arXiv:hep-ph/0103349].
- [28] V. Agrawal, S.M. Barr, J.F. Donoghue and D. Seckel, “Anthropic considerations in multiple-domain theories and the scale of electroweak symmetry breaking,” *Phys. Rev. Lett.* **80**, 1822 (1998) hep-ph/9801253.
V. Agrawal, S.M. Barr, J.F. Donoghue and D. Seckel, “The anthropic principle and the mass scale of the standard model,” *Phys. Rev.* **D57**, 5480 (1998) hep-ph/9707380.
- [29] A. Vilenkin, “Predictions From Quantum Cosmology,” *Phys. Rev. Lett.* **74**, 846 (1995) [arXiv:gr-qc/9406010].
J. Garriga and A. Vilenkin, “A prescription for probabilities in eternal inflation,” *Phys. Rev. D* **64**, 023507 (2001) [arXiv:gr-qc/0102090].
- [30] J. F. Donoghue, “The weight for random quark masses,” *Phys. Rev. D* **57**, 5499 (1998) [arXiv:hep-ph/9712333].
- [31] G. Aldazabal, S. Franco, L. E. Ibanez, R. Rabadan and A. M. Uranga, “Intersecting brane worlds,” *JHEP* **0102**, 047 (2001) [arXiv:hep-ph/0011132].
- [32] L. E. Ibanez, F. Marchesano and R. Rabadan, “Getting just the standard model at intersecting branes,” *JHEP* **0111**, 002 (2001) [arXiv:hep-th/0105155].
- [33] D. Cremades, L. E. Ibanez and F. Marchesano, “Standard model at intersecting D5-branes: Lowering the string scale,” *Nucl. Phys. B* **643**, 93 (2002) [arXiv:hep-th/0205074].
- [34] D. Cremades, L. E. Ibanez and F. Marchesano, “Towards a theory of quark masses, mixings and CP-violation,” arXiv:hep-ph/0212064.

- [35] D. Cremades, L. E. Ibanez and F. Marchesano, “Yukawa couplings in intersecting D-brane models,” JHEP **0307**, 038 (2003) [arXiv:hep-th/0302105].